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Regime transitions in the viscous catenary

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Abstract – We present studies of the transition between two types of filament evolution in the viscous catenary. In particular we examine the dependence of the observed behavior on the viscosity of the fluid, the initial radius of filament created, and the total length of the fluid. The kinematic viscosity is varied over two orders of magnitude, the filament radius by one order of magnitude, and the filament length by a factor of 3. We observe a clear transition as the so-called catenary number, dependent on the filament radius and length, increases past a certain value, but with no dependence on the viscosity of the fluid. This is a confirmation of theoretical predictions over a significantly greater range of viscosities than previous experiments.

Introduction. – Fifty years ago, Barnes and Woodcock, intrigued by the peculiar effects observed in filaments of high-viscosity fluids, published an article on the "liquid rope-coil effect" [1]. More recently, the problem of the evolution of the viscous catenary, a filament of high-viscosity liquid strung horizontally between two endpoints, has received attention, starting with theoretical work by Teichman and Mahadevan [2]. The problem resembles the classical catenary but is, of course, not in static equilibrium. Major experimental studies of the problem were completed in a B.A. thesis and further study at Pomona College [3,4] supported by important theoretical work in France by de Gennes and Brochard-Wyart [4,5] as well as suggestions by Clanet in [4] (Appendix A).

In the latter paper the authors present and discuss two types of behavior observed in the catenary. The first is the so-called quasi-parabolic catenary, in which surface-tension effects in the filament can be ignored. In this case the center of the filament remains thick during the evolution while the side supports stretch until an eventual rupture, as predicted by [2]. In the second type of behavior, called a "U"-shaped filament, the initial strand of fluid created has a smaller radius. Initially the filament is observed to drop, but decelerates until the center becomes almost stationary and is stretched by the tendency of the side supports to pivot towards a vertical orientation. The filament is eventually observed to break in the center.

In [4] the authors present a physical argument for the dynamics of the transition between these two catenary behaviors. They argue that the crossover can be determined from the relative values of two characteristic times: the deformation growth time and the capillary drainage time. They define the deformation growth time as $t_g = \nu^*/(9gL_0) \sim \eta/(\rho g L_0)$ and the drainage time as $t_d \equiv L_0/V \sim \eta r/\gamma$. If the fluid in the center drains towards the edges faster than the deformation can occur a "U" shaped filament will be observed. Conversely, if the drainage characteristic time is greater than the deformation growth time a quasi-parabolic catenary should form. The ratio of these characteristic times is a dimensionless number which we call the "catenary number", given by

$$\mathrm{cat} \equiv \frac{t_g}{t_d} \sim \frac{a^2}{L_0 r}, \tag{1}$$

where t_g and t_d are the growth and deformation times, respectively. a is the characteristic capillary length,

$$a = \sqrt{\frac{\gamma}{\rho g}} = 1.5\,\mathrm{mm}, \tag{2}$$

for our silicone oil. L_0 is the horizontal length of the filament and r is the initial radius. Interestingly this argument predicts that the crossover should be quite independent of the viscosity of the fluid involved.

We are aware of similar, more thorough work conducted at ESPCI by Le Merrer, Seiwert, Quéré and Clanet [6]. They present a more rigorous classification system for

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the two catenary types as well as a discussion of the thread lifetime for the "U"-shaped catenaries in addition to a discussion of the transition between catenary behaviors.

Experimental methods. – We study Clearco silicone oil of 10000, 100000, and 1000000 cSt kinematic viscosity. A silicone oil soluble red dye is used to increase contrast with the background. A small amount of oil is placed in a conic section cut into two acrylic plates, initially clamped together. Tension is applied horizontally to the plates with rubber bands. When the clamp is released the plates are abruptly pulled apart by the bands along rails. For standard data the filament created has a length of 16.7 mm. We also have a set of data where the total length of the catenary is varied up to 45 mm while the amount of fluid used is kept constant at 0.27 g. In this case we use only the 100 000 cSt oil. This procedure produces a single filament of oil anchored at the bottom of the half conic section on each plate. Stroboscopic images of the resulting filament evolution are recorded on a digital camera and analyzed with computer software.

Observations. – We characterize the behavior of filaments generated with oils of the three kinematic viscosities described above, ranging over a factor of 100, using in each case two or three oil masses, allowing us to observe both the QP and the U behavior with each type of oil. As noted in the caption of fig. 1, the observed time-evolution allows us to distinguish unequivocally between the two types of behavior (the reasons for the apparent deceleration of the U-shaped catenary are quite subtle and discussed in detail in [4]). Typical results are as shown in table 1. For each parameter entry in table 1, we generate five filaments that clearly evolve into one type of catenary. For all parameters these five filaments at that single parameter are self-consistent in the resulting catenary type, indicating reasonable stability of parameter generation. Figure 1 illustrates the typical behaviors observed.

Image analysis of the initial filament diameters reveals that the diameter varies significantly depending on the viscosity of the oil used. We also observe diameter variations at one set of mass/viscosity parameters due to variations in the speed at which the plates pull apart. Therefore, some of the discrepancies between different masses and viscosities can be attributed to differing tension forces used to pull the plates apart and form the filament. Tensions were chosen experimentally to produce the most qualitatively uniform and reproducible filaments.

Because of the significant variation in diameters, for each catenary we measure the filament diameter in the center of the catenary for the first stroboscopic flash, excluding two of the thirty-five total in which successive stroboscopic exposures overlap and mask the initial diameter. This is plotted in fig. 2, as well as the data with varied lengths (100000 cSt viscosity oil). Errors are estimated as a number of pixels uncertainty in the original images for the edges of the length and radius. Filaments with a high catenary number have a small radius, which has a high relative uncertainty, and this is the reason for the large errors for high catenary number data.

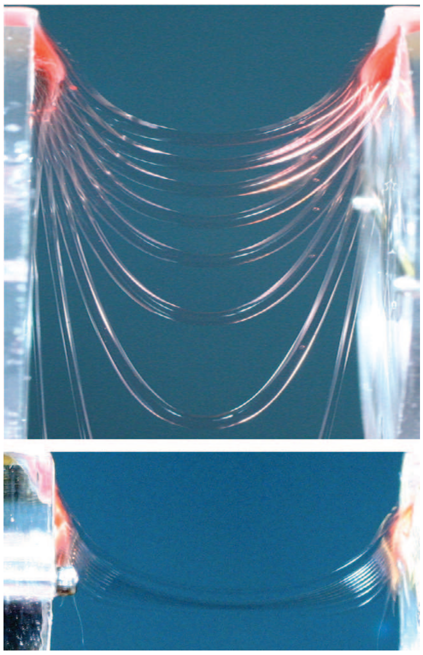

Fig. 1: (Colour on-line) Top: quasi-parabolic catenary, formed from 0.27 g of 10 000 cSt viscosity oil, the interval between strobe flashes is 0.10 s. Bottom: "U"-shaped catenary, formed from 0.05 g of 10 000 cSt viscosity oil (strobe flash interval 0.15 s). The bottom image has been contrast enhanced with computer software. It should be noted that the QP catenary displays continuous acceleration, while the U-shaped one appears to decelerate. It is this characteristic that allows us to distinguish unequivocally between the two classes of behavior.

Table 1: Behaviors observed for various initial masses and viscosities. U = "U"-shaped catenary, QP = quasi-parabolic catenary.

Viscosity (cSt)	Mass (g)		
	0.05 g	0.1 g	0.27 g
10000	U	QP	QP
100000		U	QP
1000000		U	QP

We clearly observe a transition at approximately cat $\sim$ 0.5. Above approximately 0.5 we only observe "U"-shaped catenaries and below only the quasi-parabolic. We do observe a small amount of overlap of points around the transition, but a sharp transition is consistent with the error bars of the data. From the data points and error bars near the transition, we can place bounds on the value of the catenary number at the transition for this data to cat $= 0.51 \pm 0.04$.

Conclusions. – We note that, in fig. 2, the observed transition is evidently a function of the catenary number, but is clearly not dependent on kinematic viscosities even

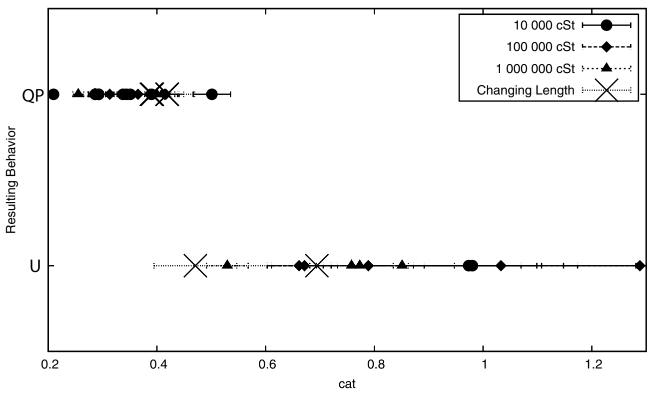

Fig. 2: Two-state diagram of the behavior observed at various catenary numbers. Fluid viscosities are identified by symbols. Data for varying catenary lengths is identified by an $\times$.

though these range over a factor of 100. We therefore conclude, as the major new experimental result of this letter that the final state into which the catenary evolves is indeed independent of the viscosity of the fluid, and is governed solely by the catenary number, as predicted by previous theoretical arguments. We have verified this when the catenary number is changed by varying the length of the filament as well. We observe the transition between the QP and U regimes at $\mathrm{cat} = 0.51 \pm 0.04$, consistent with the theoretical prediction of a transition when the growth and deformation times are comparable.

* * *

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